Fermilab
Accelerator Division
Linac Department

In Situ Calibration of Beam Position Monitors

Fermilab Report - Linac Upgrade Note 215

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June 3, 1993

I. Overview:

Bench-top calibration of Beam Position Monitors (BPMs) for the Fermilab Linac Upgrade have shown a typical offset of 0.003 inches between the electrical center of the BPM and its mechanical center. At the location of each BPM is a quadrupole magnet of the focussing system. The aperature of the quadrupole is large enough to allow a displacement from the center of the quad of up to .040 inches. Alignment of the four quadrupoles on a girder will lead to accuracies of less than .005 inches through a module. Module-to-module alignment errors will be on the order of .01 inches.[1] With such a number of possible sources of error, it is thus desirable to determine an in-line calibration technique to accurately determine the offset in the BPM reading when the beam position is through the magnetic center of the quadrupole. This note will look at the possibility of developing such a technique for the Fermilab Linac Upgrade.

II. Effect of Quadrupole Strength on Beam Position:

Figure #1 shows a schematic diagram of the BPM positions in the Linac Upgrade. With each BPM being located inside of a quadrupole magnet, the first thought would be to look at the effect of the quad strength upon position of the beam through the quad. Thus we consider a particle with coordinates (X_0,X_0') which goes through a thin lens followed by a drift space L. This drift space would be the distance from one BPM to the next. The position that we read from the BPM at this place (X_1) is:

$$X_1 = (1 \pm Lf)\, X_o + L\, X_o' + X_{1offset} \tag{1}$$

where + sign denotes defocussing lens and - sign is for a focussing lens; $X_{1offset}$ is the unknown offset in the next uncalibrated BPM; and f is focal length given by:

$$\frac{1}{f} = K\ell = \frac{qB'\ell}{mc\beta\gamma} \tag{2}$$

with ℓ being the length of the lens. After a bit of substitution, equation (1) becomes:

$$X_1 = (\pm L\ell X_o)K + (LX_o' + X_o + X_{1offset}) \tag{3}$$

Therefore, a plot of X_1 versus K should produce a straight line with slope of $\pm L\ell X_o$. Since L and ℓ are both known we thus find X_o. Comparing this with the value read from the BPM provides us with the calibration offset that we are looking for. Continuing this procedure down the linac enables all BPMs to be calibrated.

III. Thin lens? :

Of course the first question that must be asked when using a thin lens approximation is "Is it a good approximation?" The equation for the position of a particle at a drift distance L after a focussing quadrupole without using the thin lens approximation is:

$$X_1 = \left(\cos\sqrt{K}\ell - \sqrt{K}L\sin\sqrt{K}\ell\right)X_o + \left(\frac{1}{\sqrt{K}}\sin\sqrt{K}\ell + L\cos\sqrt{K}\ell\right)X_o' \tag{4}$$

which is certainly more complex than Equation 3.

To examine the effects of these two calculations, let's assume some reasonable numbers such as $X_o = 2$ mm, $X_o' = 2$ mradians. At the entrance to the first accelerating module of the Linac Upgrade, L=155.5 cm and $\ell = 8.5$ cm from Tom Kroc's calibration of the quadrupoles.[2] Using a spreadsheet to calculate values of X_1 in the region ±25% of the design value of $K = 13.3\ m^{-2}$ and then plotting X_1 versus K yields a slope of $-2.76 \times 10^{-4}\ m^3$ whereas from the simple thin lens model a slope of -2.65×10^{-4} is expected. This is a 4% discrepancy.

A similar calculation done at the exit of the upgrade where L = 242.8 cm and the nominal value of $K = 6.83\ m^{-2}$ gives a slope of $-4.34 \times 10^{-4}\ m^3$ while the thin lens calculation yields $-4.14 \times 10^{-4}\ m^3$, a discrepancy of 4.6%.

Looking more closely at the terms in Equation 4:

$K\ (m^{-2})$	$\cos\sqrt{K}\ell$	$\sqrt{K}L\sin\sqrt{K}\ell$	$\frac{1}{\sqrt{K}}\sin\sqrt{K}\ell$	$L\cos\sqrt{K}\ell$
nominal (13.3)	0.952	1.73	0.084	1.48
+25% (16.6)	0.941	2.15	0.083	1.46
-25% (10.0)	0.964	1.31	0.084	1.50

Over this range, the sine function is linear and the cosine function is approximately constant. Therefore, to within an uncertainty of 4-5% the thin lens equation allows a much simpler analysis for the calibration of BPMs.

III. Error Analysis:

Since slope = ± L ℓ X_0 the error in our determination of X_0 is:

$$\frac{\sigma_{X_0}^2}{X_0^2} = \frac{\sigma_s^2}{s^2} + \frac{\sigma_L^2}{L^2} + \frac{\sigma_\ell^2}{\ell^2} \tag{5}$$

The measured effective length of the quadrupoles is 8.53 ± 0.10 cm, a relative error of 1.2%. Assuming an error in the distance between BPMs of a few millimeters, the relative error in L is on the order of a few tenths of a percent since the distance between BPMs is large (155.5 cm at entrance to linac upgrade, 243 cm at exit).

To estimate the error in the measurement of the slope of X_1 vs K, measurements were performed on the existing linac. The strength of the final quad in tank 5 (horizontally defocussing) was varied while the beam position at the entrance to tank 6 was measured. The procedure was also repeated for the final quad in tank 6 (horizontally focussing) and the beam position at the entrance to tank 7. Values of L = 82.5 cm (distance from quad to BPM) and ℓ = 16.459 cm (quad effective length) were used to determine X_0.

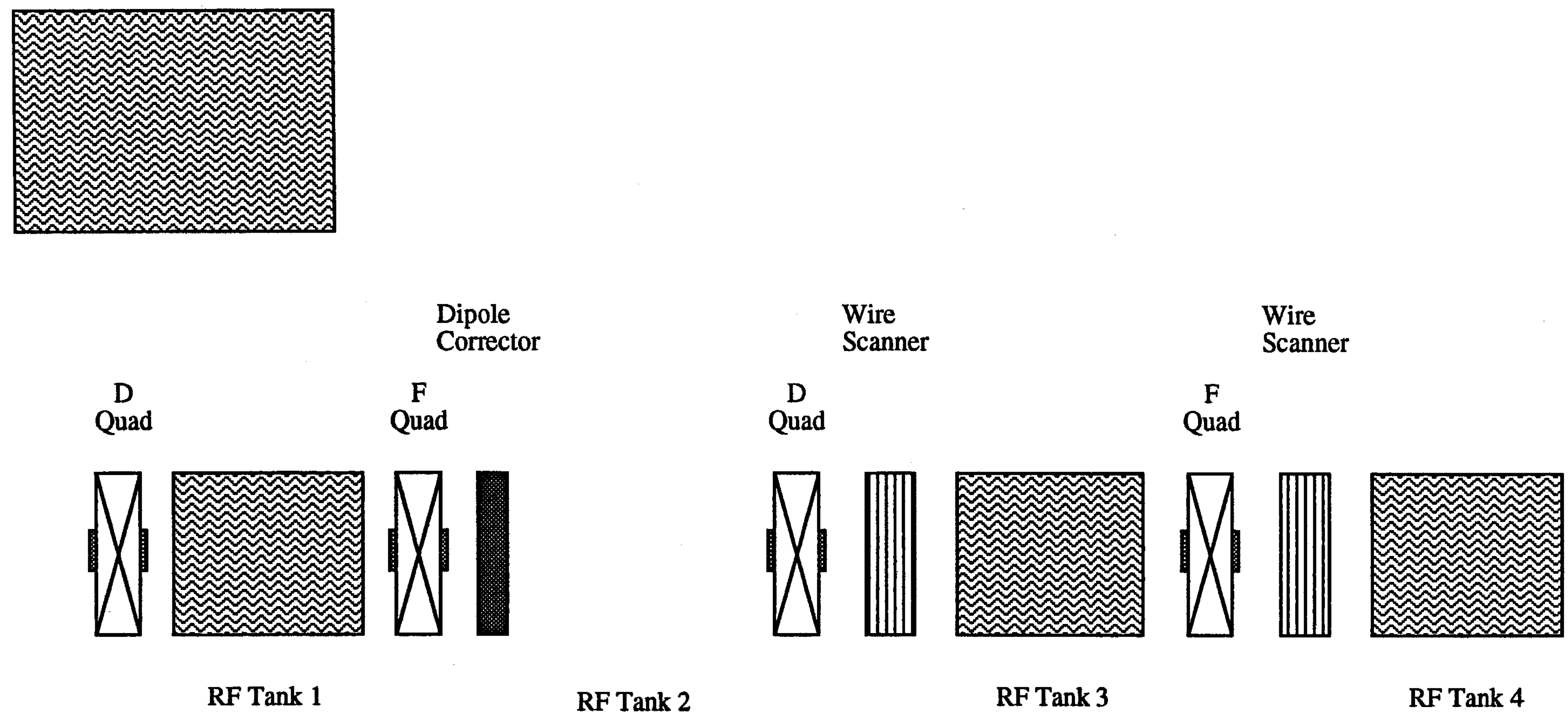
BPM	Slope of X_1 vs K (mm m^2)	X_0 (mm)
BPH6IN	0.603 ± 0.035	4.44 ± 0.26
BPV6IN	0.764 ± 0.023	-5.63 ± 0.17
BPH7IN	0.023 ± 0.015	-0.17 ± 0.11
BPV7IN	-0.096 ± 0.022	-0.71 ± 0.16

From the data we see that for offsets on the order of several millimeters, the relative uncertainty in X_0 is on the order of 5%. This is the same magnitude of uncertainty introduced by the thin lens approximation. Fortunately, the beam appears to be very close to the center of the quad at the exit of tank 6. To estimate a minimum detectable offset we can use the data from the BPM at the entrance to tank 7 where there is an uncertainty in the slope of 0.015 mm m^2. In the Linac Upgrade (L=155.5 cm and ℓ = 8.5 cm) this would translate to an uncertainty in the measurement of X_0 of 0.11 mm.

IV. References:

[1]Fermilab Linac Upgrade Conceptual Design, Nov 1989.

[2]T.K. Kroc, "Preliminary Analysis of Linac Upgrade Quad", Linac Upgrade Note.


Dipole
Corrector
Wire
Scanner
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D
Quad
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RF Tank 1
RF Tank 2
RF Tank 3
RF Tank 4

Figure #1